Decision Theory Problem

The value of research information can be assessed by several means, one of which is decision theory. The example considered here concerns the case of a manager who is deciding on a change in production equipment. Research information will play a major role in this decision. The new equipment can be leased for five years and will replace several old machines that require constant attention to operate. The problem facing the manager is, "Shall I lease the new machines with the attendant efficiencies, reduced labor, and higher lease charges, or shall I continue to use the old equipment?"

The decision situation has been prompted by news that the firm might secure several large orders from companies that have not been previous customers. With added volume, departmental profit contributions will increase substantially with the new equipment. For this decision, the manager adopts the decision variable "average annual departmental profit contribution."[1] The decision rule is, "Choose that course of action that will provide the highest average annual contribution to departmental profits."

Exhibit DT–1 indicates the results of the evaluation of the two available actions. Under the conditions cited, it is obvious that course A_1 is preferred.

Conditions of Certainty

Exhibit DT–1 presents the case with the assumption that the anticipated new business will materialize. It therefore represents, in decision theory terminology, *decision making under conditions of certainty.* It is assumed the payoffs are certain to occur if the particular action is chosen and the probability of the additional business being secured is 1.0.[2] The decision to choose action A_1 is obvious under these conditions with the given payoff data and decision rule.

Conditions of Uncertainty

In a more realistic situation, the outcome is less than certain. The new business may not materialize, and then the department might be left with costly excess capacity. The union may resist introduction of the new equipment because it replaces workers. The new equipment may not perform as anticipated. For these or other reasons, the decision maker may be uncertain about the consequences (for instance, that course A_1 will result in a $20,000 contribution).

Suppose the manager considers these other possible outcomes and concludes the one serious uncertainty is that the new business may not be forthcoming. For purposes of simplicity, one of two conditions will exist in the future—either the new business will be secured as expected (O_1), or the new business will not materialize (O_2). In the first case, the expected payoffs would be the same as in Exhibit DT–1; but if the new business is not secured, then the addition of the new equipment would give the department costly excess capacity, with fixed lease charges. The payoff table may now be revised as Exhibit DT–2.

Under these conditions, the original decision rule does not apply. That rule said, "Choose that course of action that will provide the highest average annual contribution to departmental profits."Under the conditions in Exhibit DT–2, action A_1 would be better if the new business were secured, but A_2 would be the better choice if the new business were not secured. If the decision

EXHIBIT DT–1 Payoff Under Conditions of Certainty

Course of Action	Average Annual Departmental Profit Contribution
A_1—Lease new equipment	$20,000
A_2—Retain old equipment	12,000

EXHIBIT DT–2 Payoff Under Conditions of Uncertainty

	Average Annual Departmental Profit Contribution		
Course of Action	New Business (O_1)	No New Business (O_2)	Expected Monetary Value
A_1—Lease new equipment	$20,000	$5,000	$14,000
A_2—Retain old equipment	12,000	9,000	10,800

can be delayed until the new order question is resolved, the dilemma is escapable. However, because of lead times, the equipment decision may need to be made first.

When faced with two or more possible outcomes for each alternative, the manager can adopt one of two approaches. First, the likelihood that the company will receive the new business cannot be judged. Even so, a rational decision can be made by adopting an appropriate decision rule. For example, "Choose that course of action for which the minimum payoff is the highest." This is known as the *maximum criterion* because it calls for maximizing the minimum payoff. In Exhibit DT–2, the minimum payoff for alternative A_1 is shown as $5,000, and the minimum payoff for A_2 is $9,000. According to the *maximum rule,* the choice would be A_2 because it is the best of the worst outcomes. This decision is a "cut your losses" strategy.

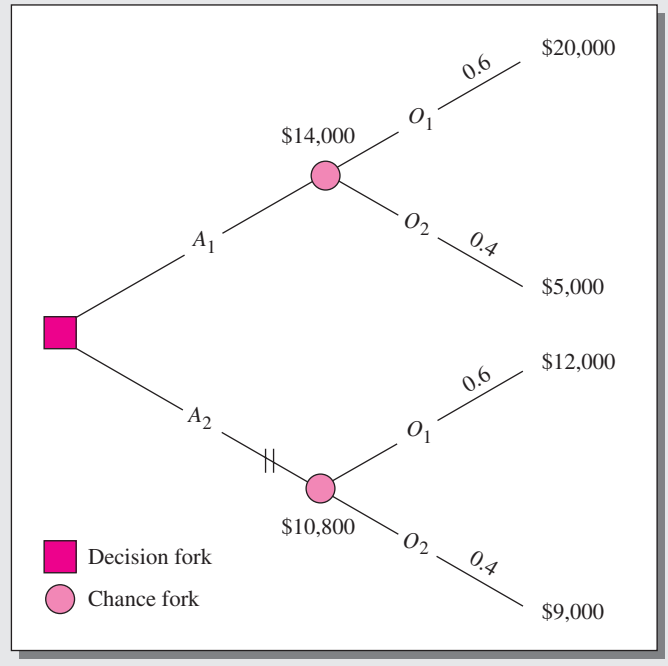
The second approach is to use subjective judgment to estimate the probability that either O_1 or O_2 will occur.[3] When the assumption was decision under certainty, only one event was possible (had a probability of 1.0). Now, however, with experience and information from other sources, there is a less-than-certain chance of the new business materializing, and this doubt should be part of the decision.

One might estimate that there is a 0.6 chance the new business will be secured and a 0.4 chance it will not. With this or any other set of similar probabilities, an overall evaluation of the two courses of action is possible. One approach is to calculate an *expected monetary value (EMV)* for each alternative.[4]

The Decision Flow Diagram

The decision problem already has been summarized in a payoff table, but further illustration in the form of a decision flow diagram (or decision tree) may be helpful. The decision tree for the equipment problem is shown in Exhibit DT–3. The diagram may be seen as a sequential decision flow. At the square node on the left, the manager must choose between A_1 and A_2. After one of these actions, a chance event will occur—either the new business will be received by the company (O_1), or it will not be received (O_2). At the right extremity of the branches are listed the conditional payoffs that will occur for each combination of decision and chance event. On each chance branch is placed the expected probability of that chance event occurring. Keep in mind that these are subjective probability estimates by the manager that express a degree of belief that such a chance event will occur.

EXHIBIT DT–3 Decision Tree for the Equipment Problem



Having set up this series of relationships, one calculates back from right to left on the diagram by an *averaging out and folding back* process. At each decision juncture, the path that yields the best alternative for the decision rule is selected. Here the EMV for A_1 averages out to $14,000, while the EMV for A_2 is $10,800. The double slash line on the A_2 branch indicates it is the inferior alternative and should be dropped in favor of A_1.

The Contribution of Research

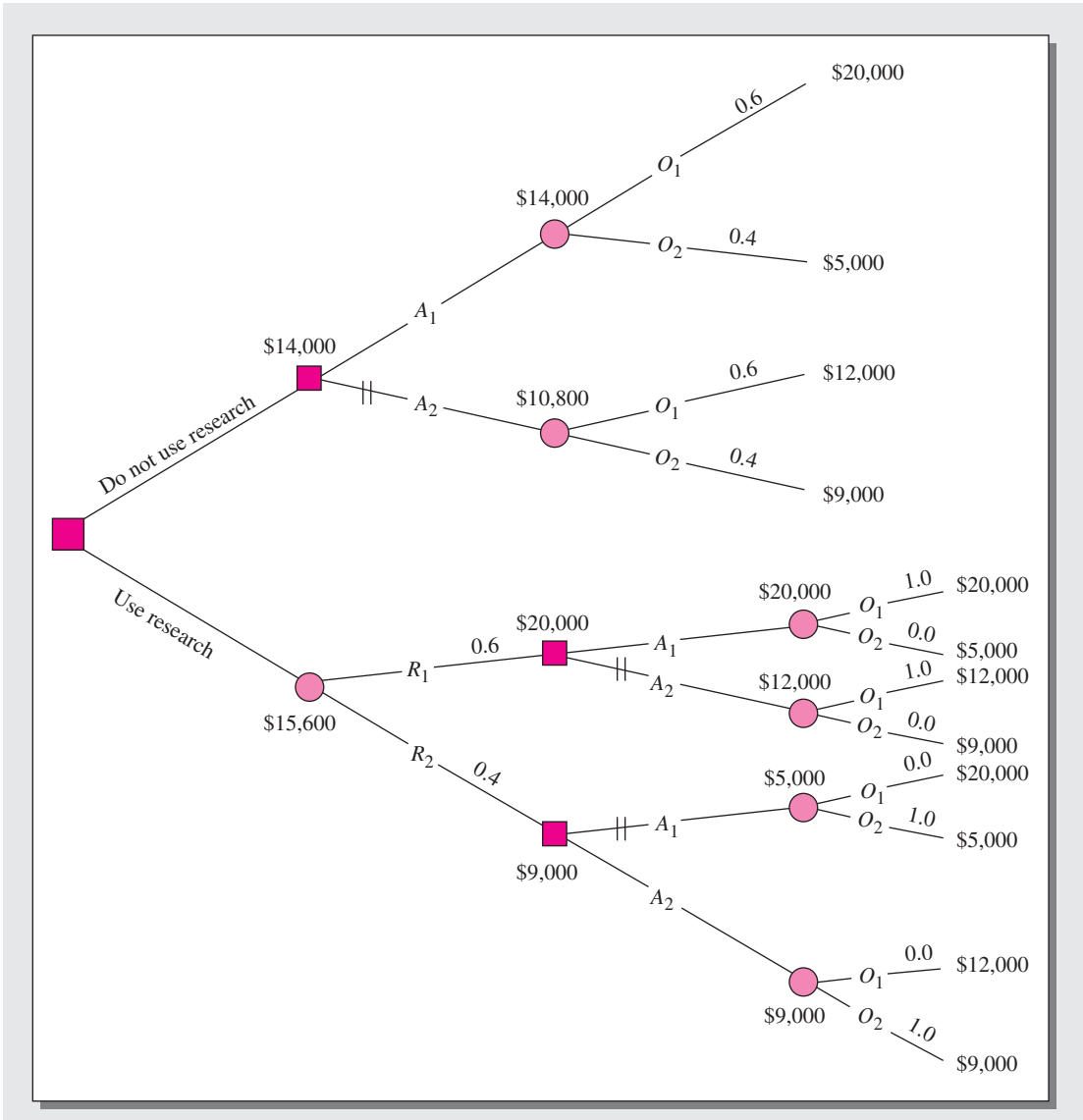
Now the contribution of research can be assessed. Recall that the value of research may be judged as "the difference between the results of decisions made with the information and the results of decisions that would be made without it." In this example, the research need is to decide whether the new business will be secured. This is the uncertainty that, if known, would make a perfect forecast possible. Just how much is a perfect forecast worth in this case?

Consider Exhibit DT–3 once again. What would happen if the manager had information to accurately predict whether the new business orders would be secured? The choice would be A_1 if the research indicated the orders would be received, and A_2 if the research indicated the orders would not be received. However, at the decision point (before the research is undertaken), the best estimate is that there is a 0.6 chance that the research will indicate the O_1 condition and a 0.4 chance that the condition will be O_2. The decision flow implications of the use of research are illustrated in Exhibit DT–4.

The decision sequence begins with the decision fork at the left. If the manager chooses to do research (R), the first chance fork is reached where one of two things will occur. Research indicates either that the orders will be received (R_1) or the orders will not be received (R_2). Before doing the research, the best estimate of the probability of R_1 taking place is the same as the estimate that O_1 will occur (0.6). Similarly, the best estimate that R_2 will occur is 0.4.

After the manager learns R_1 or R_2, there is a second decision fork: A_1 or A_2. After the A_1-A_2 decision, there is a second chance fork (O_1 or O_2) that indicates whether the orders were received.

EXHIBIT DT-4 The Value of Perfect Information



Note that the probabilities at O_1 and O_2 have now changed from 0.6 and 0.4, respectively, to 1.0 and 0.0, or to 0.0 and 1.0, depending on what was learned from the research. This change occurs because we have evaluated the effect of the research information on our original O_1 and O_2 probability estimates by calculating *posterior probabilities.* These are revisions of our prior probabilities that result from the assumed research findings. The posterior probabilities (for example, $P(O_1|R_i)$ and $P(O_2|R_i)$ are calculated by using Bayes's theorem.[5]

The manager is now ready to average out and fold back the analysis from right to left to evaluate the research alternative. Clearly, if R_1 is found, A_1 will be chosen with its EMV of $20,000 over the A_2 alternative of $12,000. If R_2 is reported, then A_2 is more attractive. However, before the research, the probabilities of R_1 and R_2 being secured must be incorporated by a second averaging out. The result is an EMV of $15,600 for the research alternative versus an EMV of $14,000 for the no-research path. The conclusion then is this: Research that would enable the manager to make a perfect forecast regarding the potential new orders would be worth up to

$1,600. If the research costs more than $1,600, decline to buy it because the net EMV of the research alternative would be less than the EMV of $14,000 of the no-research alternative.

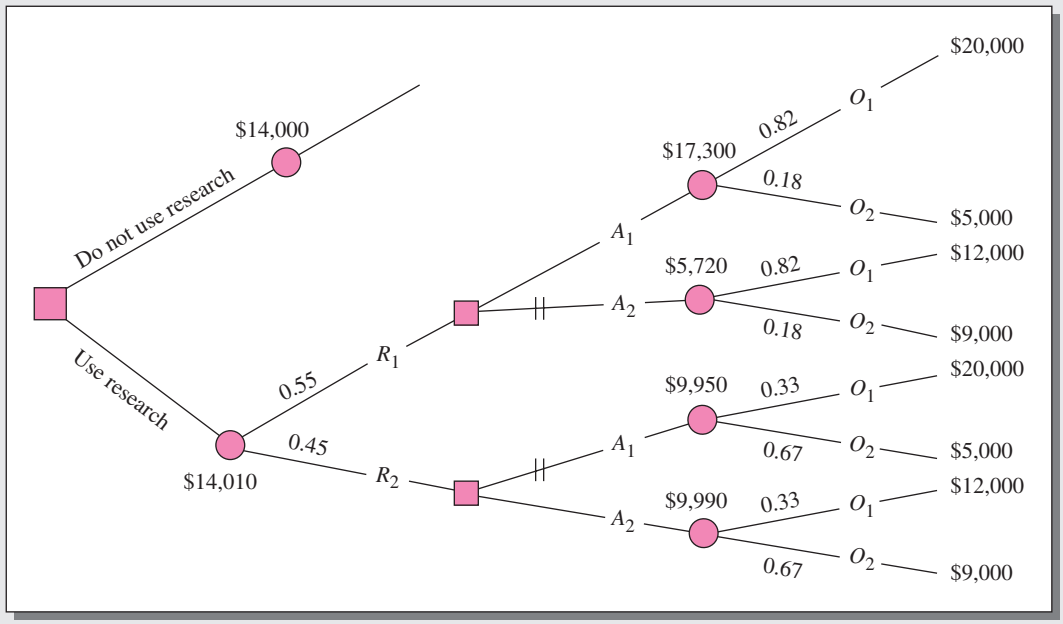
	States of Nature			Posterior Probabilities	
Research Outcomes	O_1	O_2	Marginal Probabilities	$P(O_1\|R_i)$	$P(O_2\|R_i)$
R_1	0.6	0.0	0.6	1.0	0.0
R_2	0.0	0.4	0.4	0.0	1.0
Marginal probabilities	0.6	0.4			

Imperfect Information

The analysis up to this point assumes that research on decision options will give a perfect prediction of the future states of nature, O_1 and O_2. Perfect prediction seldom occurs in practice. Sometimes research reveals one condition when later evidence shows something else to be true. Thus, we need to consider that the research in the machinery decision will provide less-than-perfect information and is, therefore, worth less than the $1,600 calculated in Exhibit DT–4.

Suppose the research in that example involves interviews with the customers' key personnel and some customers' executives. They might all answer our questions to the best of their ability but still predict imperfectly what will happen. Consequently, we might judge that the chances of their predictions being correct are no better than 3 to 1, or 0.75. If we accept that our research results may provide imperfect information in this manner, we need to factor this into our evaluation decision. We do this by averaging out and folding back again. The results are shown in Exhibit DT–5. The revised EMV, given research judged to be 75 percent reliable, is $14,010. This

EXHIBIT DT–5 The Value of Imperfect Information



revised EMV is only $10 higher than the $14,000 EMV using no research and would seem to be hardly worth consideration.

Pragmatic Complications

This discussion, while simplified, contains the basic concepts for finding the value of research. Practical difficulties complicate the use of these concepts. First, the situation with two events and two alternatives is artificial. Problems with more choices and events are common, and the chief complication is the increased number of calculations.

	States of Nature			Posterior Probabilities	
Research Outcomes	O_1	O_2	Marginal Probabilities	$P(O_1\|R_i)$	$P(O_2\|R_i)$
R_1	0.45	0.10	0.55	0.82	0.18
R_2	0.15	0.30	0.45	0.33	0.67
Marginal probabilities	0.60	0.40			

A more serious problem is posed by the measurement of outcomes. We have assumed we could assess the various actions in terms of an unambiguous dollar value, but often we cannot. It is difficult to place a dollar value on outcomes related to morale or public image, for example.

An allied problem lies in the exclusive use of EMV as the criterion for decision making. This is correct in an actuarial sense and implies that each decision maker has a linear system of evaluation. In truth, we often use another evaluation system. The person who accepts EMV as a criterion sees that an even bet of $20 between two people on the toss of a fair coin is a fair bet. Many people, however, may not be willing to make such a bet because they fear the loss of $20 more than they value the gain of $20. They may need to be offered a chance, say, to win $20 but to lose only $10 before they would be willing to bet. These persons have a nonlinear decision scale. The "utility" concept is more relevant here.

The development of more precise methods of evaluating the contribution of research continues. In the meantime, continued emphasis on the improvement of our understanding of the researcher's task and the research process will make research more valuable when it is conducted.

Reference Notes

1. Recall that the decision variable is the unit of measurement used in the analysis. At this point, we need not be concerned with how this measure is calculated or whether it is the appropriate decision variable. Assume for purposes of this illustration that it is appropriate.
2. A probability is a measure between 1.0 and 0.0 that expresses the likelihood of an event occurring. For example, the probability of a "head" on a toss of a coin is 0.5. Under conditions of certainty, the forecasted outcome is assumed to have a probability of 1.0 even though we might agree that we normally cannot know the future with certainty. In most forecasting where a specific amount is named, there is an implicit assumption of certainty.
3. Concepts of probability enter into three types of situations. In the classical situation, each possible outcome has a known chance of occurrence. For example, a coin tossed in the air has a 0.5 chance of landing heads up; a spade card has a 0.25 chance of being drawn from a well-mixed deck.

   In the same type of situation, probabilities are thought of as "relative frequencies." Even if the probability is not known from the structure of the problem (as it is in the classical case), it can still be estimated if there is a body of empirical evidence. For example, experience may show that about 1 in 50

products produced is defective. From this statistic, one can estimate there is a 0.02 chance that any given product will be defective.

If there is no direct empirical evidence, one can still assess probability on the basis of opinion, intuition, and/or general experience. In such cases, uncertainty is expressed as a subjectively felt "degree of confidence" or "degree of belief" that a given event will occur. The discussions in this appendix are cases in point. For more information on probability concepts, see any modern statistics text.

4. One calculates an EMV for an alternative by weighting each conditional value (for example, \$20,000 and \$5,000 for A_1) by the estimated probability of the occurrence of the associated event (0.6 probability of the \$20,000 being made).

$$\begin{aligned} EMV &= P_1(\$20{,}000) + P_2(\$5{,}000) \\ &= 0.6(\$20{,}000) + 0.4(\$5{,}000) \\ &= \$14{,}000 \end{aligned}$$

5. Bayes's theorem with two states of nature is

$$\begin{aligned} P(O_1|R_1) &= \frac{P(R_1|O_1) \times P(O_1)}{P(R_i|O_1) + P(O_1) \times P(R_i|O_2) \times P(O_2)} \\ &= \frac{1.0 \times 0.6}{(1.0 \times 0.6) + (0.0 \times 0.4)} \\ &= 1.0 \end{aligned}$$